Chaos and Self-Organization

Bio202
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Population Growth

Simplest model:

$N_{(i+1)}=RN_i$

R>1 implies $\lim_{i \to \infty} N_i = \infty$

R=1 $\lim_{i \to \infty} N_i = 1$

0<R<1 $\lim_{i \to \infty} N_i = 0$

Resources are finite

Populations cannot grow infinitely

Population Growth with Carrying Capacity

$N_{i+1}=R(N_i)$
$R=r(1-K^{-1}N_i)$
r=birth rate, K=carrying capacity

$N_{i+1}=rN_i\,(1-K^{-1}N_i)$

Dimensionless variable
$x_i=K^{-1}N_i$

$x_{i+1}=rx_i(\,1 - x_i\,)$

Predictions

Does the population level reach a steady state value?

Is it dependent on initial value (x_0)?

Does it depend on r?

Can the population become extinct?

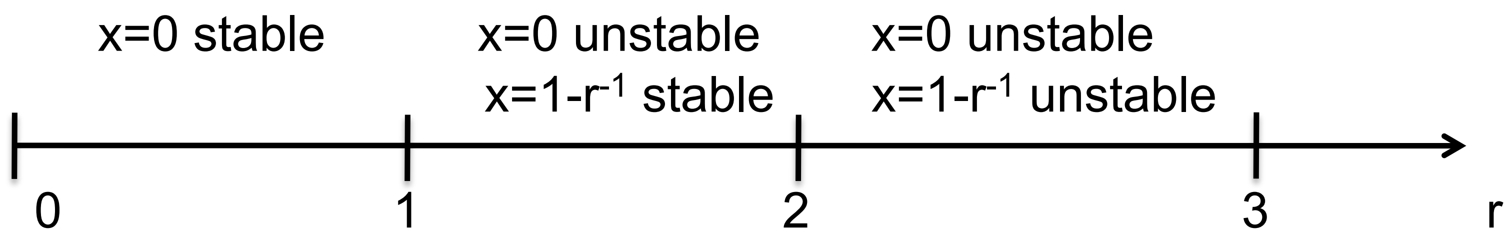
Does the population ever oscillate?

Demonstration

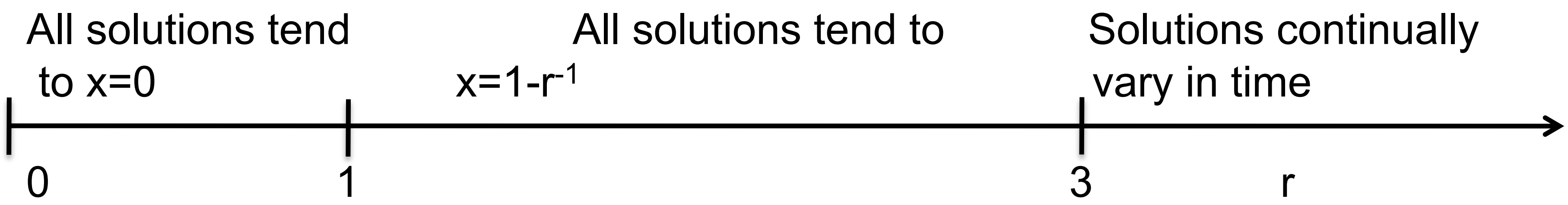
Difference equation

$$x_{i+1}=rx_i(\ 1 - x_i\ )$$

Steady State Analysis



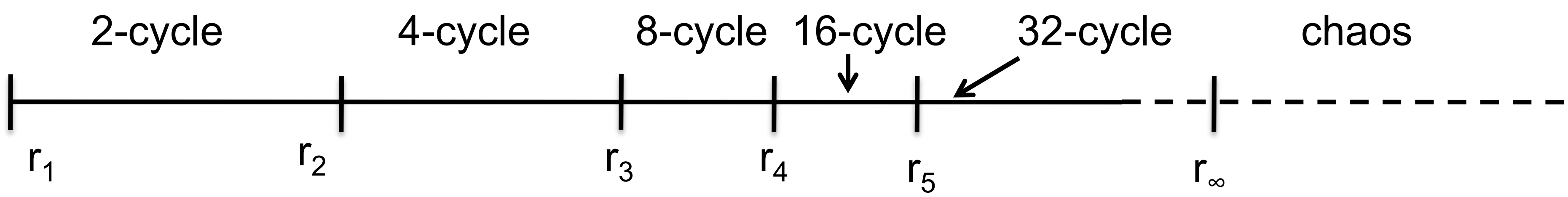
Qualitative Behaviour



May R. (1974,1975)

Onset of Chaos

$r_1=3$
$r_2=3.449$
$r_3=3.544$
$r_\infty=3.570$



For $r>3$, no stable steady solution

$3<r<3.499$ solution of period $n=2$

$3.544<r<3.57$ succession of transitions $n=18, 16, 32,\ldots$ Period doubling

$R>3.57$ solution depends critically on initial conditions

Chaos

Poncaire (1903) "small differences in initial conditions produce very great ones in the final phenomena".

Lorenz (1963) Deterministic non-periodic flow. J. Atmos. Sci. 20:130-141.

Li & Yorke (1975)Period three implies chaos. Am.Math. Monthly. 82: 985-992.

May R.M. (1976) Simple mathematical models with very complicated dynamical behaviour. Nature 269: 459-467.

Deterministic equations, with even slightly different initial conditions ($1/10^3$) lead to dramatic differences in predicted time courses

Definition of Chaos

Aperiodic

Bounded

Deterministic

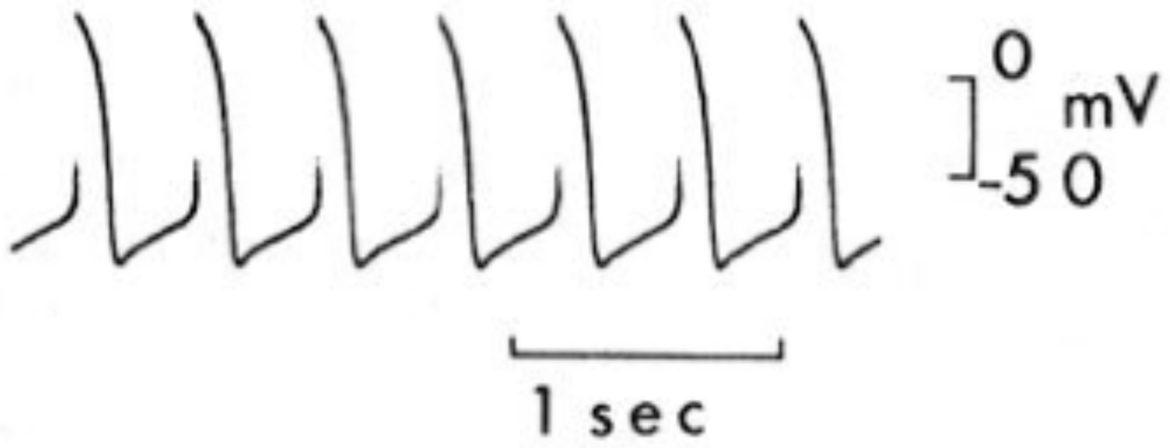
Sensitive dependence on initial conditions

Chaos in Cardiac Myocytes

Chicken embryonic ventricular heart cells cultured

Cells spontaneously begin to contract

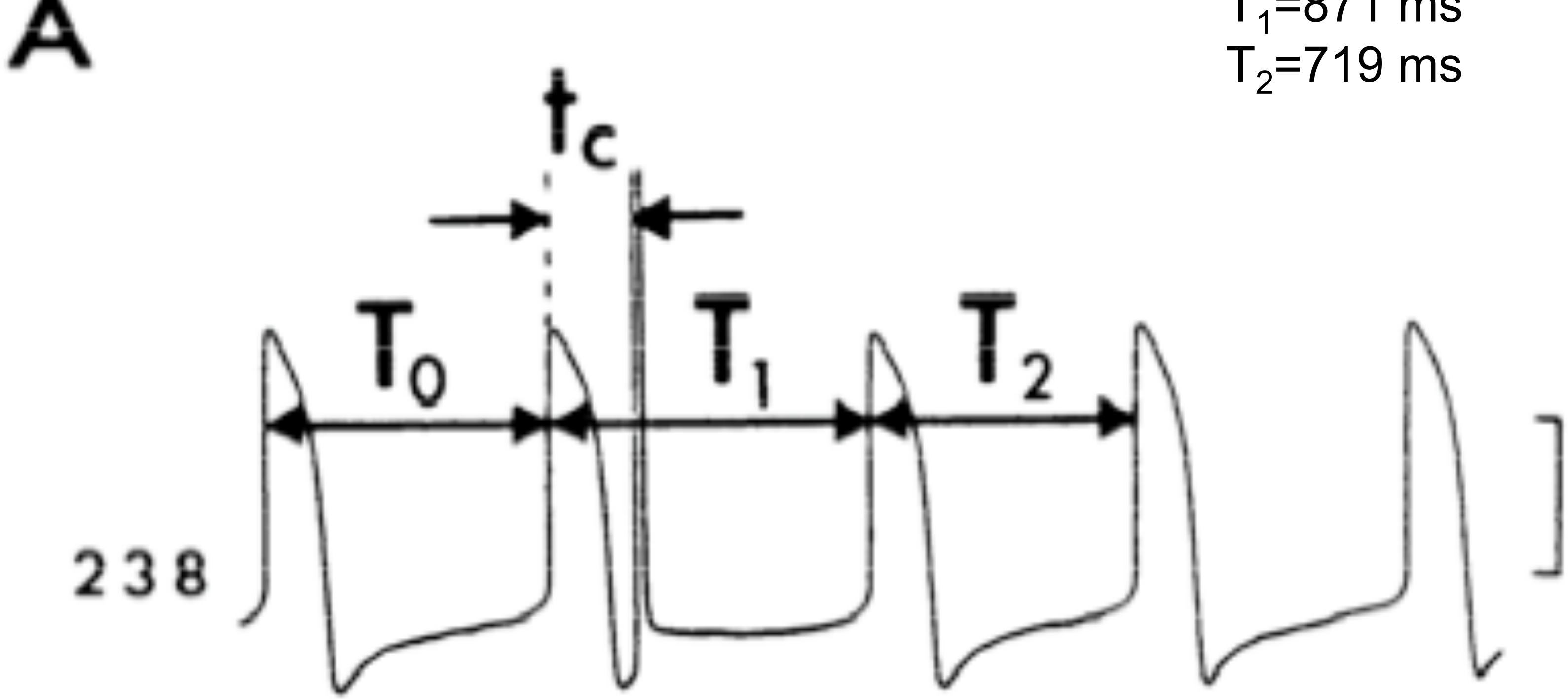
Measured electrical activity- action potential



Guevara et al. (1986)

Phase Resetting by Depolarizing Current

T_0=768 ms
t_c=238 ms
T_1=871 ms
T_2=719 ms



Chaos from Periodic Stimulation

Finite difference equation

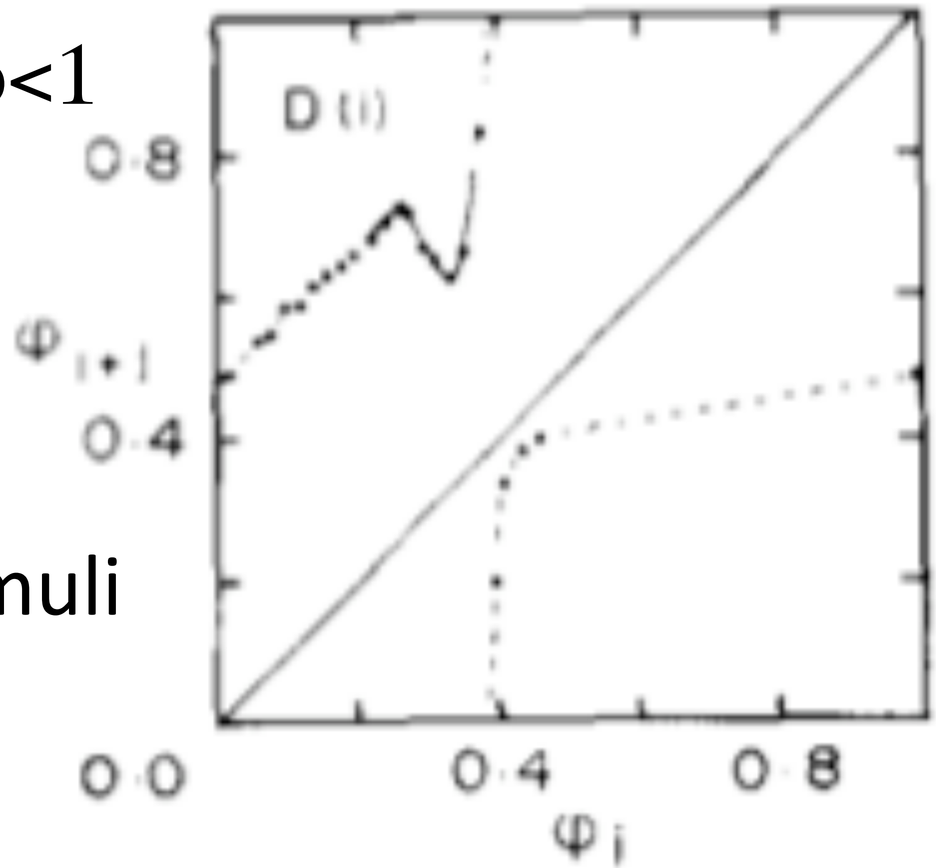
Phase $\phi = t_c/T_0$, $0 \leq \phi < 1$

$\phi_{i+1} = g(\phi_i) + \tau \pmod 1$

τ=interval between stimuli

ϕ=phase

$g(\phi)$= new phase



- - - - - - - theory

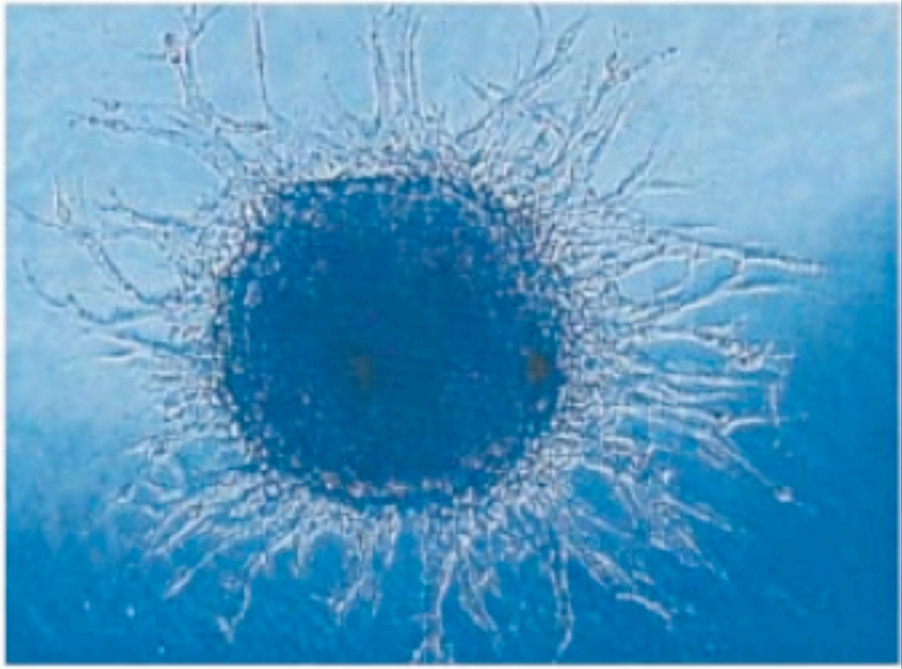
▪ ▪ ▪ ▪ ▪ ▪ experiment

References

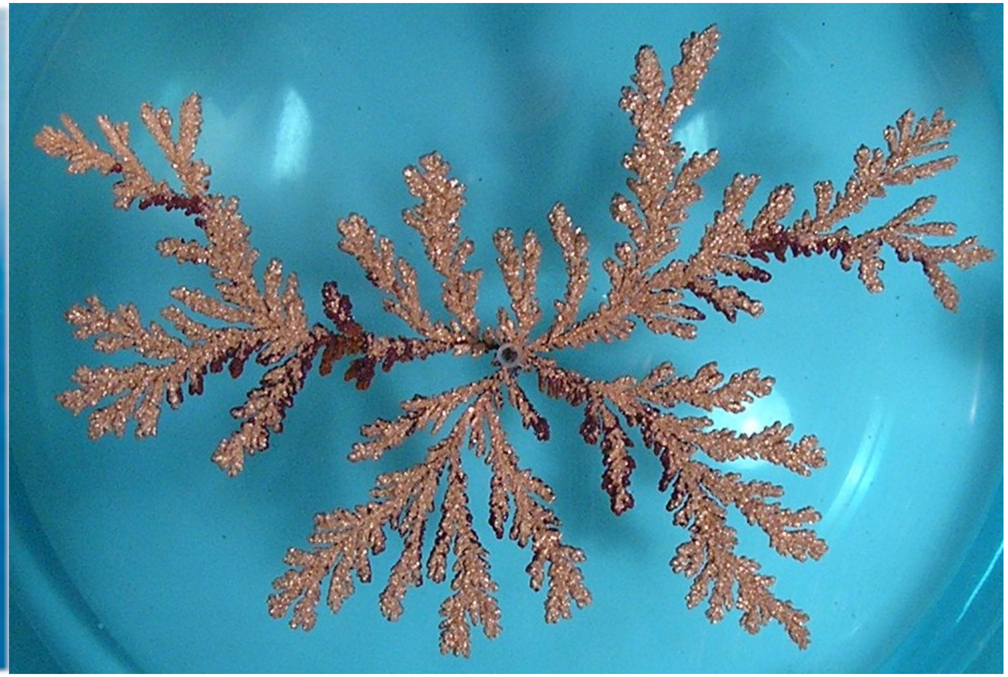
Modelling dynamic phenomena in molecular and cellular biology- Segel L.A.

Understanding Nonlinear Dynamics- Kaplan & Glass

Self Organization



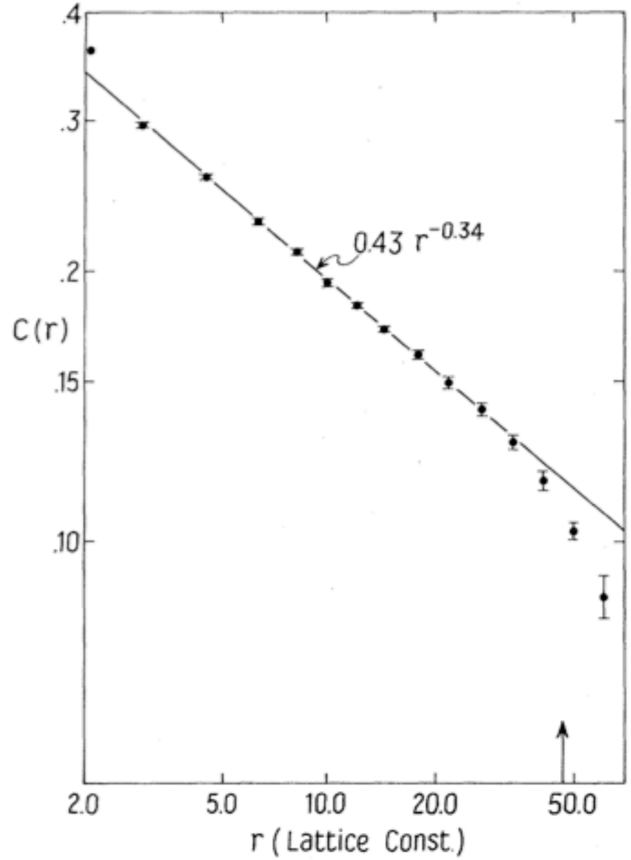
Deisboeck et al. (2001)



Wikipedia

Diffusion Limited Aggregation

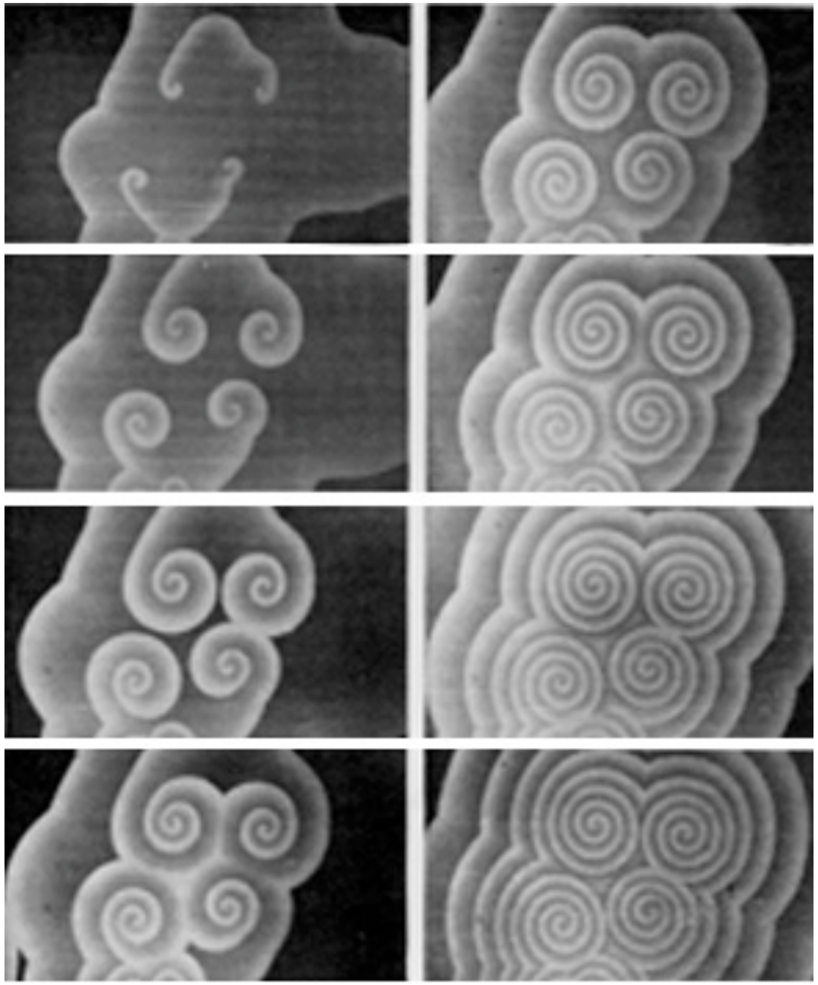
FIG. 1. Random aggregate of 3600 particles on a square lattice.



$$C(r) = N^{-1} \sum_{r'} \rho(r')\rho(r'+r),$$

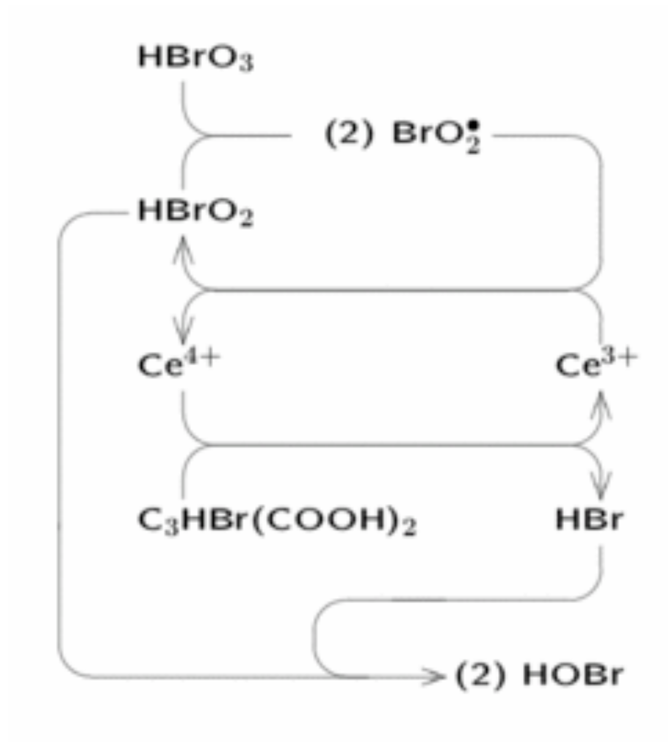
Witten & Sander (1981)

B-Z Reaction

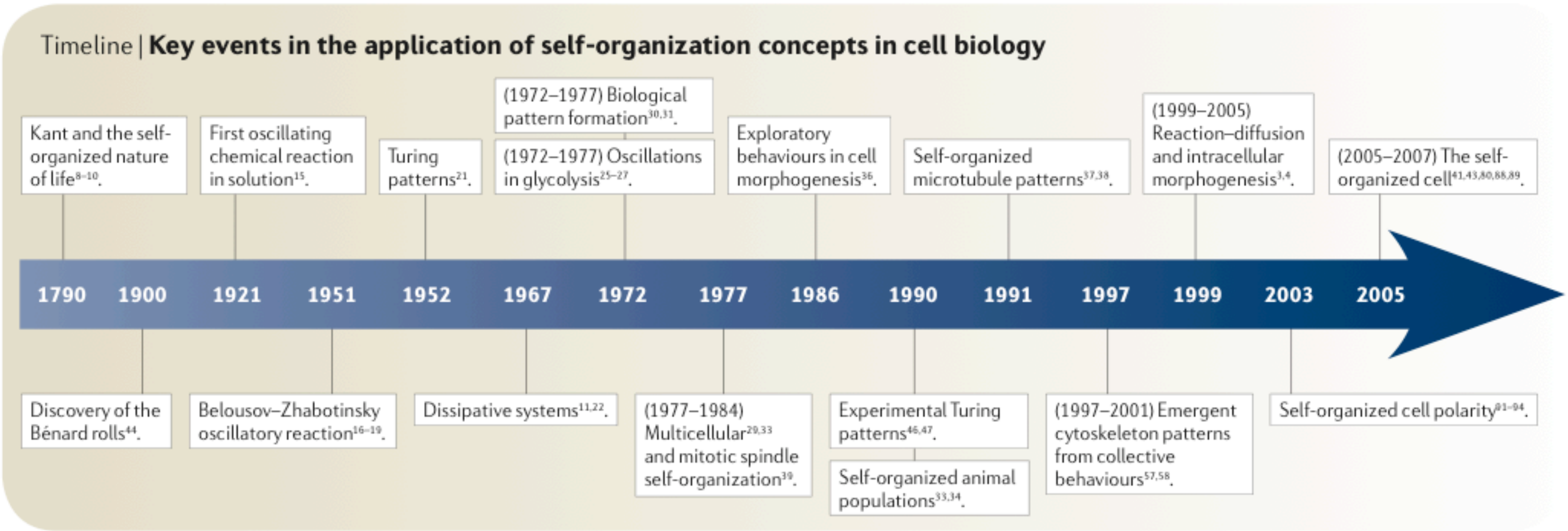


Zhabotinsky

Zhabotinsky and Chaikin (1971)



Historical Timeline

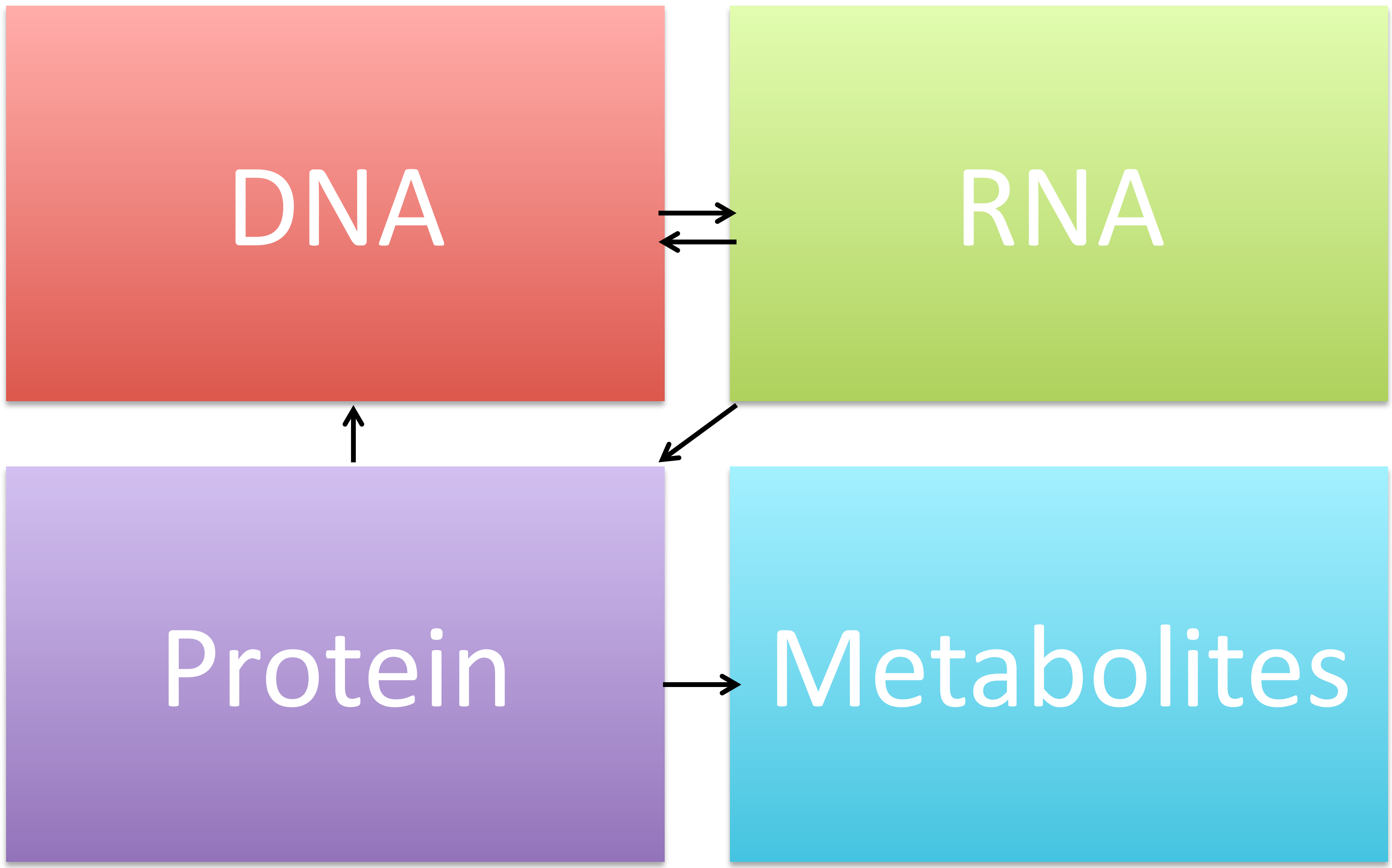


Self-organization: The dynamic organization that emerges from the collective behaviour of agents, the individual properties of which cannot account for the properties of the final dynamical pattern.

Template vs. Self-Organization

Template	Self-Organization
Rules complex, global	Rules simple, local
Equilibrium	Out of equilibrium
Structure apparent from component behaviour	System property non-intuitive property of whole
Eg. Central dogma of molecular biology, self-assembly	Eg. DLA, Turing patterns, non-linear dynamics, feedback loops

Central Dogma



Self Organization in Biology: Landmarks

D'arcy Wentworth Thompson (1917) On growth and form.

Alfred J. Lotka (1910) The theory of autocatalytic chemical reactions.

Vito Volterra (1926) A statistical analysis of fish catches in the Adriatic

Turing (1952) 'The chemical basis of morphogenesis' from *Philosophical Transactions of the Royal Society of London*, Series B, No.641, Vol. 237.

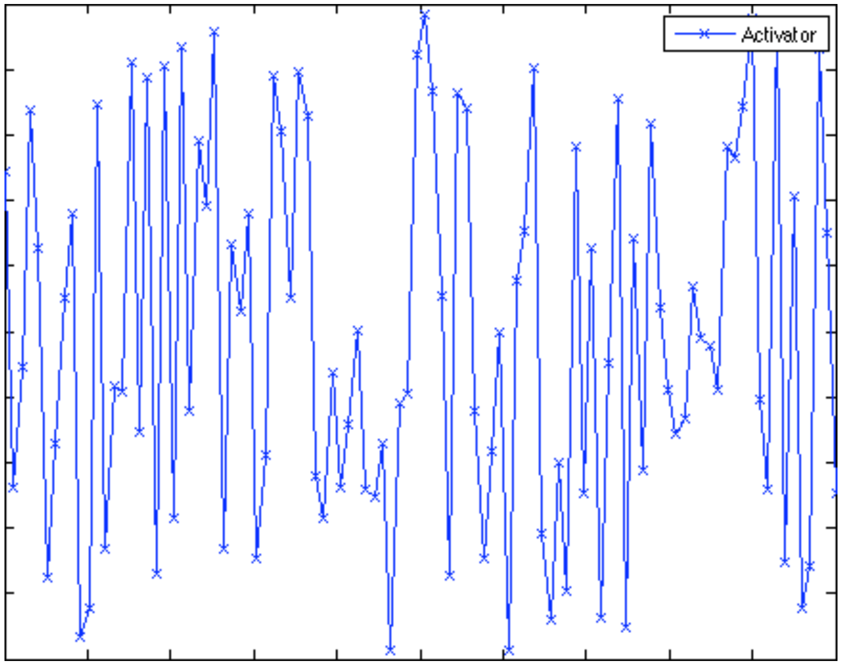
1 Turing pattern

Turing Pattern

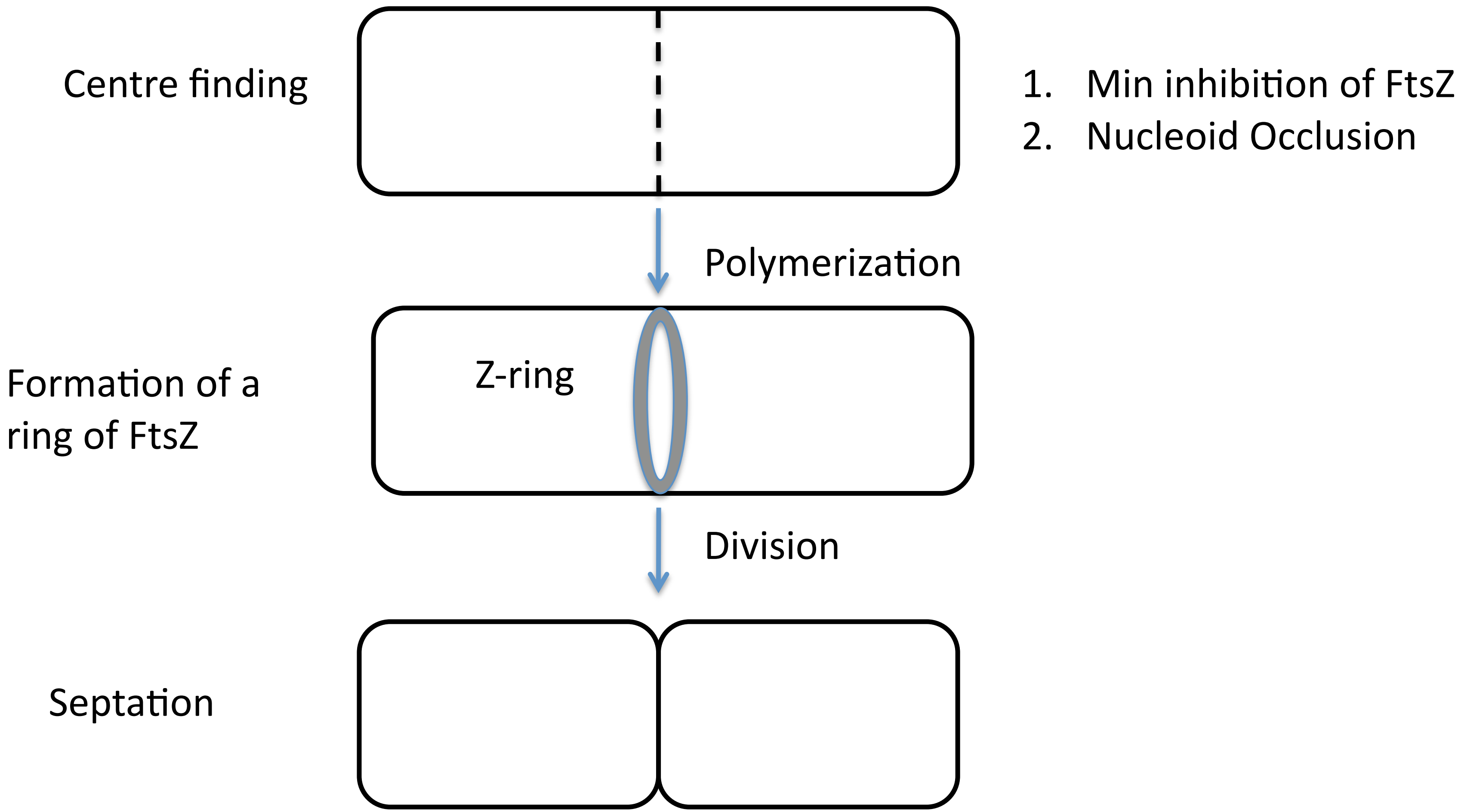
$$\frac{\partial[A]}{\partial t} = g \cdot [A] - f \cdot [I] - [A]^3 + D_a \cdot \Delta[A] \qquad (1)$$

$$\frac{\partial[I]}{\partial t} = h \cdot [A] - j \cdot [I] + D_i \cdot \Delta[I] \qquad (2)$$

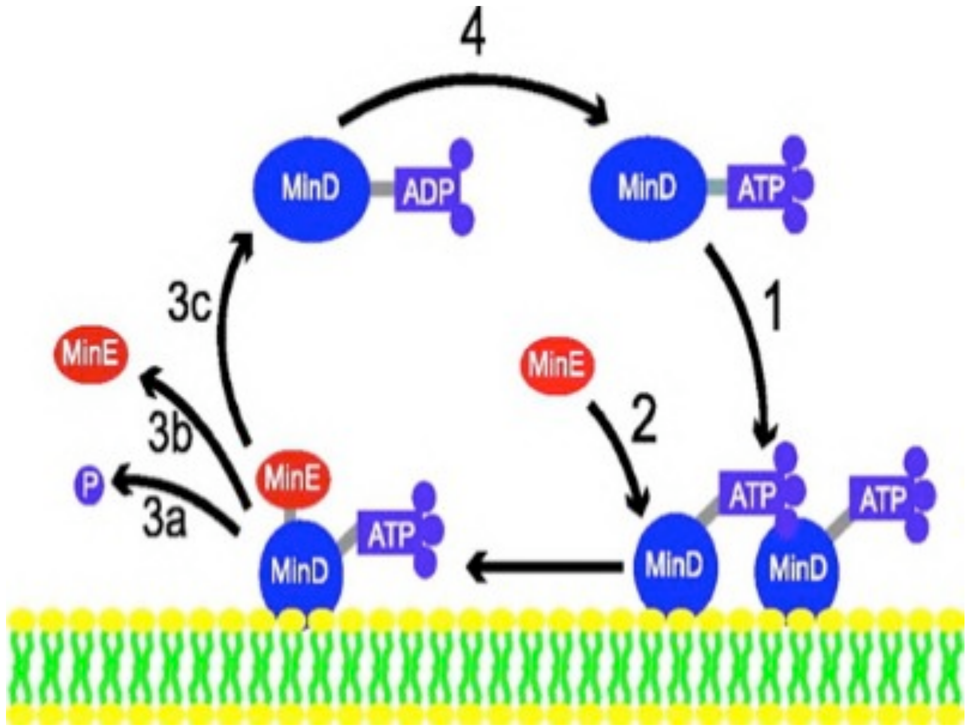
$D_i / D_a = 50$



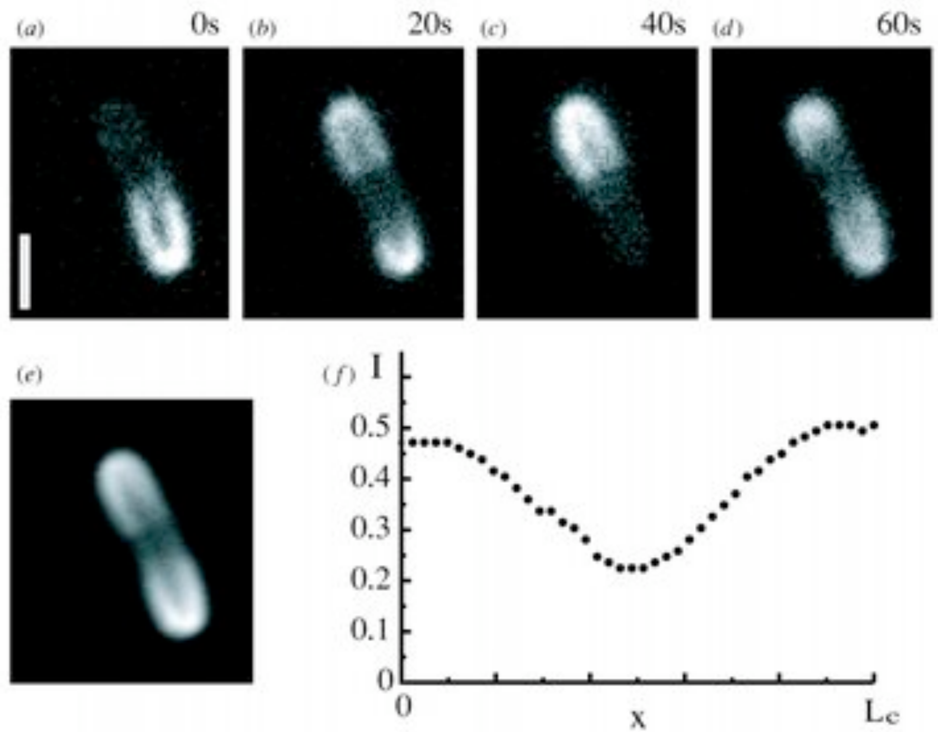
Finding the Centre in E. coli



Finding the Centre



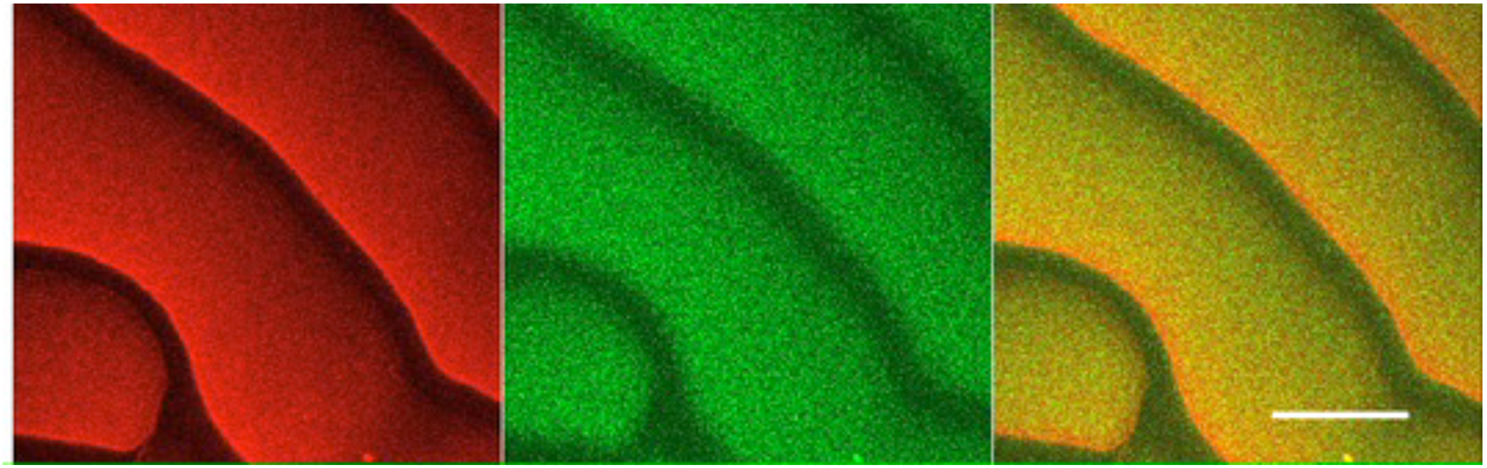
The Min cycle



MinD inhibits FtsZ polymerization

Meacci G. et al. (2006) Phys Biol. 3, p255

Diffusion Reaction Patterns



MinE MinD Merge

Min protein waves.
MinD (1 µM), doped with 20% Bodipy-labeled MinD (green), MinE (1 µM), doped with 10% Alexa647-labeled MinE (red).

Scale bar is 50 µm, frames are 9 s apart

Loose M, et al (2008) Science, 320, p789

Next

Mathematical modeling and Biology (3)

a. Theories for Signal transduction (1)
b. Developmental biological models (2)

Properties of Chaotic Systems

Sensitive dependence on initial conditions

Grahically: loss of neighbourhood in terms of location

x(t) time-series diverges

Any small difference explodes exponentially

Exact information about system equations, limited precision of present conditions snowball over time

Initial conditions if known with infinite precision, minimal errors in computation will destroy long-term prediction